

## **Numerical verification on application of SVD to SHM for bridge structure**

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### **ABSTRACT**

SHM (Structure Health Monitoring) for bridge structures has been intensively studied, recently. The vibration-based SHM uses natural frequency and mode shape as indices. If free or stationary vibrations are obtained, the estimation of these modal parameters can be easily carried out. The free vibrations and micro-tremors, however, are very small and it makes the vibration-based SHM difficult. On the other hand, the traffic-induced vibrations are large enough to measure accurately, at the low cost. However, there are technical issues on using traffic-induced vibrations to SHM on how to analyze and on how to estimate the modal parameters from the transient responses. The aim of this study is to find a new method to estimate the mode shapes from the data of an output-only system, without the input information. SVD (Singular Value Decomposition) is focused on and is applied as the objective time is changing. It is found that SVD can estimate the mode shapes of a bridge structure in stable accuracy.

### **BACKGROUND**

The traditional inspection method for civil structures generally means visual check. Its efficiency depends on skill of veteran engineers. The number of veteran engineers, however, is predicted to run short soon, because of a large number of degrading bridges. The vibration-based SHM (Structure Health Monitoring) technology has been expected to be a solution to overcome this problem. The vibration indices never change due to the engineer's skill.

The technical issues of vibration-based SHM, however, are still discussed between many researchers. One of the issues is the accuracy of damage detection. The natural frequency is the most popular index. This index can be expressed by the mass and flexural stiffness of the structure. Thus, the changes of the natural frequencies often indicate the structural changes. However, while temperature and humidity often influence on the whole structural system and make the natural frequencies change, the bridge damages rarely do. The reason why the bridge's damage does not change its frequencies lies in its locality. A bridge damage like a crack significantly effects changes on the structural performance at the limited area, while its influence is averaged and mostly disappeared in the system parameters such as the natural frequencies. Some researchers focus on the spatial index such as the mode shape. The bridge's vibration mode shapes can be described by the spatial distribution of the mass and flexural stiffness. Because of that, the mode shape often shows the high sensitivity to the local damages. Although the calculation of mode shapes requires the data measured at the several points on the structure, it is one of the cost-effective approaches. As another technical issue, there is the problem of non-stationary signal. If you analyze the stationary signals, you can accurately estimate the modal parameters such as the natural frequencies and mode shapes. The excitation of the stationary vibration, however, is very difficult on a bridge. The predominant vibration components are generally traffic-induced vibration on the middle span bridges, and this is non-stationary or in other word transient. It is generally cut out from the scoped data, while the amplitude is much more significant than free vibrations and other micro-tremors. If it can be used for the damage verification through the signal processing, the further improvement of vibration-based SHM can be done. Thus, some researchers also focus on the time-frequency analysis. STFT (Short Time Fourier's Transformation) and CWT (Continuous Wavelet Transformation) are popular as the time-frequency analysis method.

According to the previous studies, both of the spatial analysis and the time-frequency analysis have high feasibility for the application of SHM. However, there is no examination about the hybrid analysis combines them. The purpose of this study is to propose the simple hybrid method and to examine the efficiency of it by numerical simulation.

## THEORETICAL BASIS

To estimate the mode shapes of the bridge, SVD (Singular Value Decomposition) method is applied in this study. SVD is one of the most popular orthogonal decomposition method as well as eigenvalue decomposition. The application of SVD method to the mode shape estimation means the assumption of the un-correlation between the different modal orders of basis coordinates. The un-correlation is always satisfied by the free vibrations and stationary vibration such as the micro-tremors, while the transient vibrations such as traffic-induced vibrations do not always. However, in the previous studies, this assumption is often satisfied by the traffic-induced vibrations and SVD method has easily and accurately shown the bridge's mode shapes from the traffic-induced vibrations. In this study, SVD method is applied to the each short-time signals extracted from the bridge vibration data measured while the several vehicles travel over the monitored bridge. The appropriate time range and over-rap ratio should be discussed, but it is fixed the certain value here.

The SVD of the matrix  $\mathbf{A}$  is defined as the follow.

$$\mathbf{A} = \mathbf{USV}^T \quad (1)$$

where  $\mathbf{U}$  and  $\mathbf{V}$  are orthogonal matrices and  $\mathbf{S}$  is a diagonal matrix. The decomposed matrices  $\mathbf{U}$ ,  $\mathbf{S}$  and  $\mathbf{V}$  can be uniquely determined from  $\mathbf{A}$ . If  $\mathbf{A}$  is a data matrix of bridge measured vibrations  $\mathbf{Y} = [\mathbf{y}(t_0) \dots \mathbf{y}(t_N)]$ , the decomposition result can be expressed by the following equation.

$$\mathbf{y}(t) = \mathbf{Uq}(t) \quad (2)$$

where  $\mathbf{U}$  is the estimated mode shape and  $\mathbf{q}(t)$  is the basis coordinates. The data matrix of  $\mathbf{q}(t)$  can be described as  $\mathbf{Q} = \mathbf{SV}^T$ . The production of a diagonal matrix and an orthogonal matrix shows un-correlation.

The traffic-induced vibrations do not always satisfy the assumption about  $\mathbf{q}(t)$ , SVD method is applied as changing the objective time. It can be so-called "short-time SVD method" in this study.

## NUMERICAL SIMULATION BASED ON VBI SYSTEM

By numerical simulation based on the VBI (vehicle bridge interaction) system shown in Figure 1, the short time SVD method is examined in this study. The bridge is modeled by simple finite beam elements and the vehicle is modeled by RBSM (rigid body spring model) method. The input parameters are shown in Table 1.

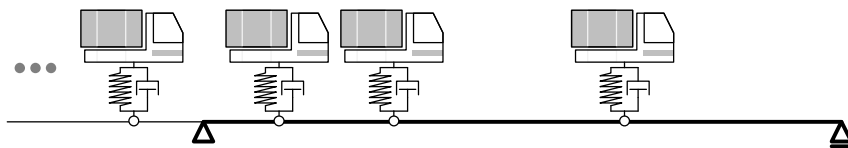


Figure 1 The diagram of numerical simulation model of VBI (vehicle bridge interaction) system

Table 1 The parameters of numerical simulation

(a) The vehicle parameters											(b) The bridge parameters		
Mass (ton)	4.0	1.9	7.9	6.6	5.8	2.4	5.0	8.7	6.4	4.9	1 <sup>st</sup> Eigen-frequency (Hz)	3.96	
Stiffness (ton/m)	157	195	116	136	193	50	219	164	34	334	Bridge length (m)	30.0	
Damping (tom/ms <sup>-1</sup> )	3.0	3.0	3.0	3.0	2.9	3.0	3.0	2.9	3.0	3.0	Flexural Stiffness (Nm)	1.56×10 <sup>10</sup>	
Eigen-frequency (Hz)	0.99	1.59	0.61	0.72	0.92	0.73	1.06	0.69	0.37	1.31	Mass per unit length $\rho$ (kg/m)	3000	
											Elements number	300	

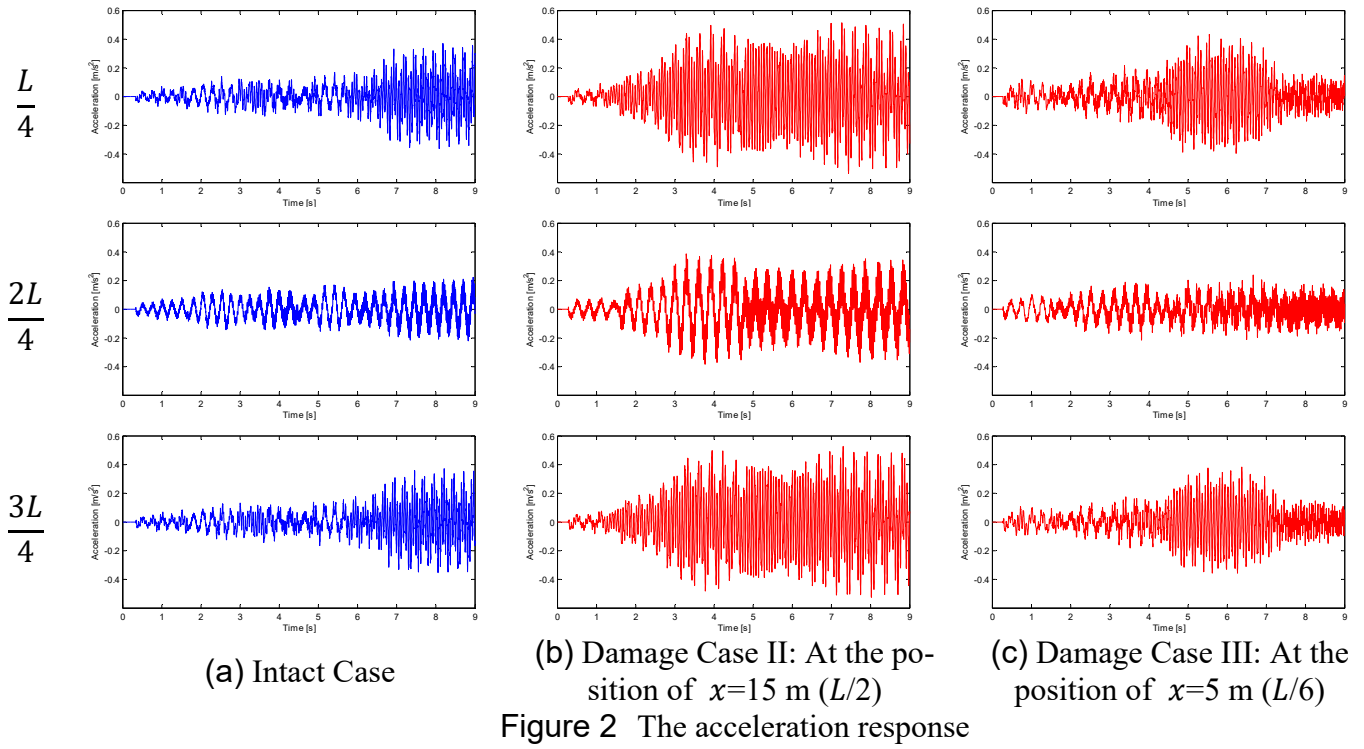


Figure 2 The acceleration response

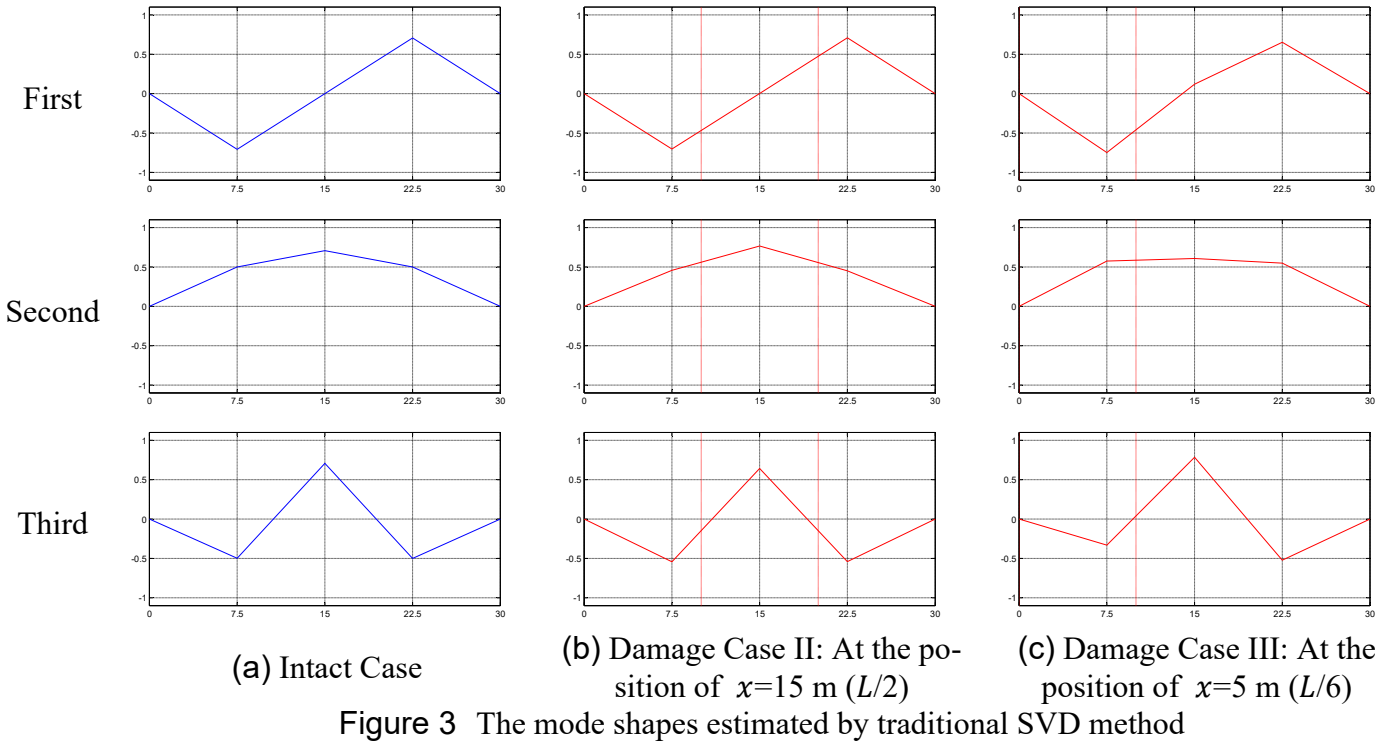


Figure 3 The mode shapes estimated by traditional SVD method

The run speed of vehicles are  $10\text{m/s}$  and the interval is  $6\text{m}$ . The vehicle mass, damping and stiffness are generated by Monte-Carlo simulation. As the damage model, the flexural stiffness is reduced  $50\%$  from the position of  $15\text{m}$  to  $25\text{m}$  in Damage Case I, and also from  $0\text{m}$  to  $10\text{m}$  in Damage Case II. Figure 2 shows the acceleration vibration responses of the bridge model. The three measuring points are set at the  $L/4$ ,  $2L/4$  and  $3L/4$ , respectively. The mode shapes estimated from these acceleration vibrations by the traditional SVD method are also shown in Figure 3. From these figures, even though the severe damage occurs, the estimated mode shapes are not so different. And, predominant mode, though it is the natural second mode shape, appears as the first mode.

## RESULTS AND DISCUSSION

Next, the results of short time SVD method are shown in Figure 4. The dividing number is 10. Thus, it means that Figure 4 indicates the first mode shapes estimated by SVD applied to the ten-equally-divided

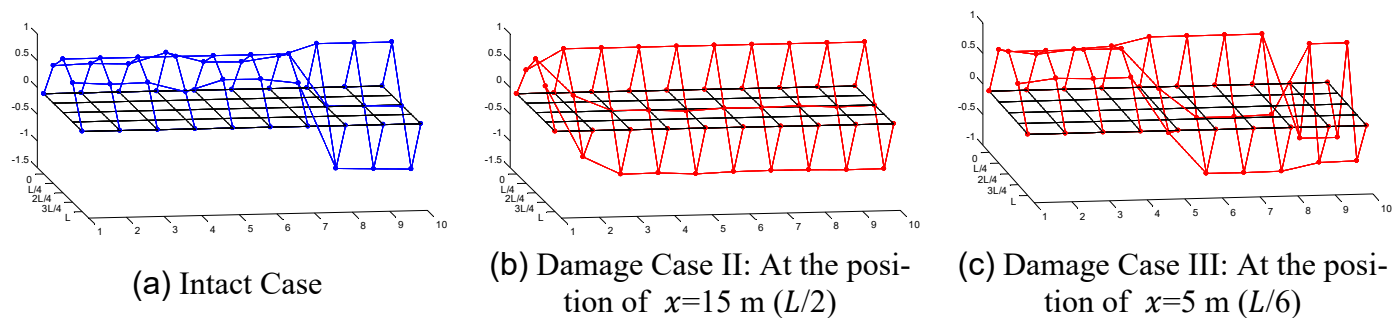


Figure 4 The estimated mode shapes by the proposed method

vibration data. The first estimated mode shapes tends to be similar to the correct first mode shape, while the high orders of mode shapes appear later. The number of divided data which gives the second or third order of mode shapes is earlier if the bridge is damaged. The reason why the second or third mode shapes are excited and estimated in this examination should be the damage effects. The damages decrease the flexural stiffness of the bridge and it makes the excitation of high order mode shapes easier. In Damage Case II, in which the flexural stiffness reduction is introduced at the edge of the span, the third mode shapes tends to easier appear.

These differences between the intact and damage cases are easier distinguished than the mode shapes estimated by the traditional SVD method. This concept is not to aim to accurately estimate the bridge natural mode shapes but to detect the signal changes due to the structure damage, even though the estimation accuracy for the modal parameters becomes lower than before.

## CONCLUSION

In this study, short-time SVD method is proposed as the time-spatial domain analysis. This method is designed to aim the improvement of sensitivity to the local bridge damage. To examine the efficiency of this method, a simple numerical simulation based on the VBI (vehicle bridge interaction) system is carried out. Although the traditional SVD method makes some errors in the estimation, the proposed short-time SVD method shows the high efficiency for the signal change detection.

This study has just examined the applicability of this method to the simple beam bridge model. The actual bridges, however, have the complicated systems in which many kinds of materials, members and structures consist. And, the numerical model of the passing vehicle is also very simple. The condition of excitation by traffic loadings is limited. The changes of the excitation tendency of the mode shapes should be affected by these factors and there are still probability that we can get the different knowledges from the numerical simulation based on the detailed models, and from the field experiments.

The simple beam tends to show the un-correlation between their basis coordinates. We will carry out the further studies about the analytical, numerical and experimental examinations about this method.

## ACKNOWLEDGEMENT

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